Eg (4) (1982)

8. Define simple harmonic motion.
The distance, x, of a particle from a fixed point, o, is given by

$$x = a\cos(\omega t + \alpha)$$

where a, ω, α are positive constants.
Show that the particle is describing simple harmonic motion about o and calculate ω and α if the velocity $v = -2a$ and $x = \frac{3a}{5}$ when $t = o$.
After how many seconds from the start of the motion is $x = o$ for the first time ?
(See Tables P.8. Take $\pi = 3{\cdot}142$).

Def^n – notes

To show particle is performing SHM
show $x = A\cos(\omega t + \alpha)$ satisfies

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$x = A\cos(\omega t + \alpha)$ A, ω, α Constant

$$\frac{dx}{dt} = -A\omega\sin(\omega t + \alpha)$$

$$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \alpha)$$

$$\frac{d^2x}{dt^2} = -\omega^2[A\cos(\omega t + \alpha)]$$

$$\frac{d^2x}{dt^2} = -\omega^2 x, \quad \text{as required}$$

To find ω and α

Told $t = o$ where $x = \frac{3}{5}A$

$\therefore x = A\cos(\omega t + \alpha)$

$$\frac{3A}{5} = A\cos(o + \alpha)$$

$$\frac{3}{5} = \cos\alpha$$

$$\Rightarrow \cos^{-1}\left(\frac{3}{5}\right) = \alpha \quad \text{(Radians!)}$$

$$\Rightarrow \alpha = .927 \text{ Radians.}$$

Next ω:

Told $v = -2A$ where $x = \frac{3}{5}A$

$$v^2 = \omega^2(A^2 - x^2)$$

$$\Rightarrow (-2A)^2 = \omega^2\left(A^2 - \left(\frac{3}{5}A\right)^2\right)$$

$$\Rightarrow 4A^2 = \omega^2\left(A^2 - \frac{9}{25}A^2\right)$$

$$\Rightarrow 4A^2 = \omega^2\frac{16}{25}A^2$$

$$\Rightarrow \frac{100}{16} = \omega^2$$

$$\Rightarrow \frac{25}{4} = \omega^2$$

$$\boxed{\frac{5}{2} = \omega}$$

To find time to reach mean position which is $\boxed{x = o}$ from initial position use given $\boxed{x = A\cos\left(\frac{5}{2}t + .927\right)}$ form as this form has the clock starting at $x = \frac{3}{5}A$.

$$\therefore \quad o = A\cos\left(\frac{5}{2}t + .927\right)$$

$$\Rightarrow \quad o = \cos\left(\frac{5}{2}t + .927\right)$$

$$\Rightarrow \cos^{-1}o = \frac{5}{2}t + .927$$

$$\Rightarrow \frac{\pi}{2} = \frac{5}{2}t + .927.$$

$$\Rightarrow \frac{\pi}{2} - .927 = \frac{5}{2}t$$

$$\Rightarrow \frac{\frac{\pi}{2} - .927}{\frac{5}{2}} = t$$

$$\Rightarrow 0{\cdot}258 \text{ seconds} = t.$$

(OR use $x = A\sin\left(\frac{5}{2}t\right)$ with $x = \frac{3A}{5}$

$$\frac{3A}{5} = A\sin\frac{5}{2}t$$

$$.6 = \sin\frac{5}{2}t$$

$$.258_{\text{sec}} = t$$
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